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Can a fast disturbance observer work under unmodeled actuators?

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Abstract: In this paper, we study robust stability of a closed-loop system with disturbance observer (DOB) when the relative degree of the plant P is greater than or equal to one, but is unknown to the control designer so that he/she chooses the nominal plant P_n of relative degree one. This situation typically happens when the actuator dynamics is ignored at the control design stage. Based on the standing assumption that the bandwidth of the low-pass Q-filter is sufficiently large, our study reveals the following results: 1) if the relative degree of P is equal to, or bigger than the relative degree of P_n by one, robust stabilization may be achieved by carefully designing the Q-filter and the nominal model P_n. It is emphasized that, when the relative degrees are not the same, not only the Q-filter but also the nominal model P_n should be carefully chosen to achieve the goal. 2) When the difference of relative degrees is greater than one, robust stabilization is not possible no matter how Q-filter and P_n are chosen. Therefore, in order to achieve robust stabilization, it is suggested to estimate the relative degree of the plant as close as possible.

Keywords: Disturbance observer, disturbance rejection, uncertainty compensation, stability, actuator dynamics.

1. INTRODUCTION

The disturbance observer (DOB) based controller has been widely used among control engineers since it has a powerful ability of uncertainty compensation and disturbance attenuation [1–7]. While there have been some research works on the stability of the DOB [4, 8–10], most of them presented somewhat conservative sufficient conditions. Recently, the authors of [11] provided an almost necessary and sufficient conditions for the DOB system to be stable, when time constant of Q-filter is chosen sufficiently small in accordance with the performance enhancement. It was shown that 1) for the closed loop stability, uncertain plant should be of minimum phase[1], and 2) for any given $C(s)$ that stabilizes the nominal plant, robust stabilization can be achieved by an appropriate choice of $Q(s)$ using the same $C(s)$. Although an almost necessary and sufficient conditions were presented in [11], it is not applicable to the case where the relative degree of real plant is not the same as that of the nominal plant.

In this paper, we study robust stability of a closed-loop system with disturbance observer (DOB) when the relative degree of the plant P is greater than or equal to one, but is unknown to the control designer so that he/she chooses the nominal plant P_n of relative degree one. This situation typically happens when the actuator dynamics is ignored at the control design stage. Based on the standing assumption that the bandwidth of the low-pass Q-filter is sufficiently large, our study reveals the following results: 1) if the relative degree of P is equal to, or bigger than the relative degree of P_n by one, robust stabilization may be achieved by carefully designing the Q-filter and the nominal model P_n. It is emphasized that, when the relative degrees are not the same, not only the Q-filter but also the nominal model P_n should be carefully chosen to achieve the goal. 2) When the difference of relative degrees is greater than one, robust stabilization is not possible no matter how Q-filter and P_n are chosen. Therefore, in order to achieve robust stabilization, it is suggested to estimate the relative degree of the plant as close as possible.

The paper is organized as follows. Section 2 introduces a basic idea of DOB controller and presents a previous result on the stability of the DOB control system. In Section 3, we present our main results that can be used to determine the stability of the DOB system when the relative degree of P is greater than (or equal to) that of P_n. Section 4 provides an illustrative example that confirms the conditions derived in Section 3. Finally, some concluding remarks are given in Section 5.

Notation: Let $D(s)$ be a polynomial with real coefficients expressed as $D(s) = d_n s^n + d_{n-1}s^{n-1} + \cdots + d_1 s + d_0$. The polynomial $D(s)$ is said to be of *degree n* if $d_n \neq 0$, which will be denoted by $\deg(D) = n$. For a transfer function $G(s) = N(s)/D(s)$ (it is assumed that $N(s)$ and $D(s)$ are coprime polynomials), the degree and the relative degree of $G(s)$ are defined as $\deg(D)$ and $\deg(D) - \deg(N)$, respectively, and the latter will be denoted by $\mathrm{r.deg}(G)$. A stable transfer function implies that its denominator is a Hurwitz polynomial. LHP (RHP, respectively) stands for the open left (right, respectively) half plane.

2. PRELIMINARY

The standard DOB configuration is illustrated in Fig. 1, which has been actively studied in, e.g., [1–4, 9, 11]. In the figure, $P(s)$ is the uncertain plant, $P_n(s)$ is its nominal model, and $C(s)$ is a controller designed *a priori* for the nominal model $P_n(s)$. The transfer function $Q(s)$ is

[1]Some extension to non-minimum phase systems, however, is presented in [12] by modifying the standard structure of the DOB and sacrificing the performance of disturbance rejection.

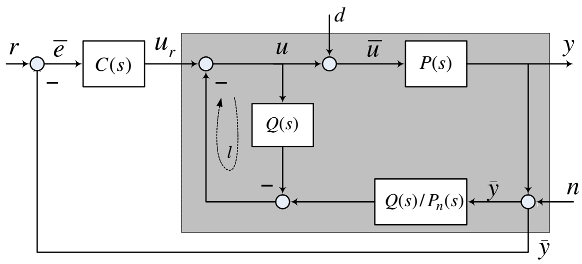

Fig. 1 The structure of the disturbance observer (DOB) controller. The shaded region represents the real plant $P(s)$ augmented with the DOB.

called a 'Q-filter', which usually has the form [2,9,11] of

$$Q(s) = \frac{c_k(\tau s)^k + c_{k-1}(\tau s)^{k-1} + \cdots + c_0}{(\tau s)^l + a_{l-1}(\tau s)^{l-1} + \cdots + a_1(\tau s) + a_0} \qquad (1)$$

where $\tau > 0$ is the filter time constant, and k and l are nonnegative integers such that $l - k$ is greater than or equal to the relative degree of $P_n(s)$. In addition, we assume $a_0 = c_0$ for the unity dc gain, *i.e.*, $Q(0) = 1$. The role of the $Q(s)$ is to make the transfer function $Q(s)P_n^{-1}(s)$ proper so that it is implementable and to avoid the algebraic loop ℓ in Fig. 1. The plant output y can be computed as

$$y(s) = T_{yr}(s)r(s) + T_{yd}(s)d(s) - T_{yn}(s)n(s), \qquad (2)$$

where

$$T_{yr} = \frac{P_nPC}{P_n(1+PC) + Q(P-P_n)},$$
$$T_{yd} = \frac{P_nP(1-Q)}{P_n(1+PC) + Q(P-P_n)},$$
$$T_{yn} = \frac{P(Q+P_nC)}{P_n(1+PC) + Q(P-P_n)}.$$

We assume that there exists an $\omega_L > 0$ such that, in the low frequency range $[0, \omega_L]$, the disturbance $d(j\omega)$ and the reference $r(j\omega)$ are significant while the noise $n(j\omega)$ is negligible. We also assume that, by choosing τ sufficiently small, Q-filter is chosen such that $Q(j\omega) \approx 1$ in the low frequency range $[0, \omega_L]$. Then, for all $\omega \in [0, \omega_L]$,

$$T_{yr} \approx \frac{P_nPC}{P + PP_nC} = \frac{P_nC}{1+P_nC},$$
$$T_{yd} \approx 0,$$
$$T_{yn} \approx \frac{P(1+P_nC)}{P+PP_nC} = 1,$$

which leads to

$$y(j\omega) \approx \frac{P_nC}{1+P_nC}(j\omega)r(j\omega), \qquad \forall \omega \in [0, \omega_L]. \qquad (3)$$

This implies that, in the frequency range $[0, \omega_L]$, the closed-loop system with the DOB behaves as if it were the nominal one in the absence of disturbance d. It should be noted that the closed-loop system must be stable for small τ in order to enjoy the virtue of the DOB controller (i.e., rejecting disturbance and compensating plant uncertainties).

The rigorous stability analysis of the closed-loop system with DOB controllers for small τ has been presented in [11], where the following facts were revealed: 1) for the closed-loop stability, uncertain plant should be of minimum phase, 2) for any given $C(s)$ that stabilizes the nominal plant, robust stabilization can be achieved solely by the choice of $Q(s)$ without changing $C(s)$. In order to introduce the condition of [11], we first let

$$p_f(s) := D_Q(s) + \left(\lim_{s\to\infty} \frac{P(s)}{P_n(s)} - 1\right) N_Q(s),$$

where $D_Q(s) = s^l + a_{l-1}s^{l-1} + \cdots + a_1 s + a_0$ and $N_Q(s) = c_k s^k + c_{k-1}s^{k-1} + \cdots + c_0$, and make the following assumption.

Theorem 1: [11] Suppose that $P(s)$ and $P_n(s)$ have the same relative degree and their high frequency gains have the same sign. Then, there exists a constant $\tau^* > 0$ such that, for all $0 < \tau \leq \tau^*$, the closed-loop system is internally stable if the following three conditions hold;
(i) $P(s)$ is of minimum phase,
(ii) $P_nC/(1+P_nC)$ is stable, and
(iii) $p_f(s)$ is Hurwitz.
On the contrary, there is $\tau^* > 0$ such that, for all $0 < \tau \leq \tau^*$, the closed-loop system is not internally stable if at least one of the conditions (i)–(iii) is violated in the sense that, $P_nC/(1+P_nC)$ has some poles in the RHP, or some zeros of $P(s)$ or some roots of $p_f(s) = 0$ are located in the RHP.

Although Theorem 1 presents an almost necessary and sufficient condition[2] for stability, it is not useful if $\lim_{s\to\infty} \frac{P(s)}{P_n(s)} = 0$ (because $p_f(s) = D_Q(s) - N_Q(s)$ has a root at the origin since $c_0 = a_0$), which occurs when the relative degree of $P_n(s)$ is less than that of $P(s)$.

3. MAIN RESULTS

Consider the closed-loop system of Fig. 1, where transfer functions from $[r, d, n]^T$ to $[\bar{e}, u, \bar{y}]^T$ are given by

$$\frac{1}{\Delta(s)} \begin{bmatrix} Q(P-P_n)+P_n, & (Q-1)PP_n, & (Q-1)P_n \\ CP_n, & (1-Q)P_n, & -Q-CP_n \\ CPP_n, & (1-Q)PP_n, & (1-Q)P_n \end{bmatrix}$$

where $\Delta(s) = (1+PC)P_n + Q(P-P_n)$. If the above nine transfer functions are stable, then the closed-loop system is internally stable. In order to analyze the stability, let P, P_n, C, and Q be represented by some ratios

[2]If some poles of $P_nC/(1+P_nC)$, or some zeros of $P(s)$, or some roots of $p_f(s) = 0$ are located on the imaginary axis in the complex plane, then Theorem 1 is not able to determine internal stability. However, if we exclude such situations, the conditions of Theorem 1 are not only sufficient but also necessary for internal stability. In this sense, they are called as an almost necessary and sufficient condition for stability.

of coprime polynomials, that is, $P(s) = N(s)/D(s)$, $P_n(s) = N_n(s)/D_n(s)$, $C(s) = N_c(s)/D_c(s)$, and $Q(s) = N_Q(s)/D_Q(s)$. Then, it has been shown in [11] that, for given $\tau > 0$, the closed-loop system is internally stable if and only if the *characteristic polynomial*

$$\delta(s) := (DD_c + NN_c)N_nD_Q + N_QD_c(ND_n - N_nD)$$

is Hurwitz.

Now, we impose a restriction under which the study is performed in this paper.

Assumption 1: The relative degree of $P_n(s)$ is equal to one and the Q-filter is of the following form

$$Q(s) = \frac{c_0}{a_1\tau s + a_0}, \qquad c_0 = a_0. \tag{4}$$

Under this assumption, the characteristic polynomial becomes

$$\begin{aligned}\delta(s) &= (DD_cN_n + NN_cN_n)a_1\tau s \\ &\quad + (NN_cN_n + D_cND_n)a_0 \\ &= a_1(\tau s)p_1(s) + a_0p_0(s),\end{aligned}$$

where

$$\begin{aligned}p_0(s) &= N(N_cN_n + D_cD_n), \\ p_1(s) &= N_n(N_cN + D_cD).\end{aligned}$$

In order to present sufficient conditions for stability, we let

$$\begin{aligned}p_0(s) &= \alpha_{m_s}s^{m_s} + \alpha_{m_s-1}s^{m_s-1} + \cdots + \alpha_0 \\ p_1(s) &= \beta_m s^m + \beta_{m-1}s^{m-1} + \cdots + \beta_0,\end{aligned}$$

where $m_s := \deg(ND_cD_n)$ and $m := \deg(N_nD_cD)$. Without loss of generality, we assume that $\beta_m > 0$. By the construction, it is seen that $m - m_s = \text{r.deg}(P) - \text{r.deg}(P_n)$.

Theorem 2: Under Assumption 1, suppose that $\text{r.deg}(P_n) = \text{r.deg}(P) - 1$. Then, there exists $\tau^* > 0$ such that, for all $0 < \tau \leq \tau^*$, the closed-loop system is internally stable if the following three conditions hold;
(i) P is of minimum phase,
(ii) $P_nC/(1 + P_nC)$ is stable, and
(iii) $\alpha_{m_s}\beta_m > 0$ and

$$\beta_m\alpha_{m_s-1} - \alpha_{m_s}\beta_{m-1} < 0. \tag{5}$$

On the contrary, there exists $\tau^* > 0$ such that, for all $0 < \tau \leq \tau^*$, the closed-loop system is unstable if at least one of the conditions (i)–(iii) is violated in the sense that some zeros of P or some poles of $P_nC/(1 + P_nC)$ are located in the RHP, or $\alpha_{m_s}\beta_m < 0$, or $\beta_m\alpha_{m_s-1} - \alpha_{m_s}\beta_{m-1} > 0$.

Proof: Since $m = m_s + 1$ from the assumption, we need to analyze $m + 1$ roots of $\delta(s) = a_1(\tau s)p_1(s) + a_0p_0(s) = 0$ as τ approaches zero. This is equivalent to study the roots of[3]

$$1 + \frac{1}{\tau}\left(\frac{a_0}{a_1}\frac{p_0(s)}{sp_1(s)}\right) = 0,$$

as $1/\tau \to \infty$. According to the standard root locus technique (e.g. [13]), as $1/\tau \to \infty$, m_s roots converge to the roots of $p_0(s)$ while the remaining $m - m_s + 1$ roots (will be denoted by s^*) to some asymptotes. By the definition of $p_0(s)$, assumptions (i) and (ii) imply that $p_0(s)$ is Hurwitz. What is left is to show that s^* is located in the LHP for small τ. To this end, let the roots of $p_0(s)$ and $p_1(s)$ be denoted by $s_{0,i}$ $(i = 1, \cdots, m_s)$ and $s_{1,i}$ $(i = 1, \cdots, m)$, respectively. Then, because[4] $\alpha_{m_s}\beta_m > 0$, it follows that s^* converges to asymptotes at angles ϕ_i radiating out from the point $s = \sigma$ on the real axis [13] where

$$\phi_i = \frac{180^\circ + 360^\circ(i-1)}{m + 1 - m_s}, \quad i = 1, 2, \cdots, m + 1 - m_s,$$

$$\sigma = \frac{\sum_{i=1}^{m} s_{1,i} - \sum_{i=1}^{m_s} s_{0,i}}{m + 1 - m_s}.$$

Since $m - m_s = \text{r.deg}(P) - \text{r.deg}(P_n) = 1$, and

$$\sum_{i=1}^{m} s_{1,i} = -\frac{\beta_{m-1}}{\beta_m}, \ \sum_{i=1}^{m_s} s_{0,i} = -\frac{\alpha_{m_s-1}}{\alpha_{m_s}},$$

s^* converges to asymptotes at angles ϕ_i from the point $s = \sigma$ where $\phi_i = 90^\circ$ or 270°, and $\sigma = \frac{1}{2}\frac{1}{\alpha_{m_s}\beta_m}(\beta_m\alpha_{m_s-1} - \alpha_{m_s}\beta_{m-1})$. From (5), it follows that $\sigma < 0$, which implies that, as τ approaches zero, s^* will reside in the LHP.

The contrary part can be shown similarly, and hence omitted. ■

Note that the first and the second conditions of Theorem 2 are the same as those of Theorem 1. On the other hand, the last condition is related with P_n, which indicates that the selection of P_n is very crucial for the stability in the case where $\text{r.deg}(P_n) = \text{r.deg}(P) - 1$. This is a key difference from Theorem 1, where the selection of P_n does not matter for $p_f(s)$ (thus, for the stability of the closed-loop system) as long as $C(s)$ stabilizes $P_n(s)$.

The following theorem shows that the closed-loop system is not stabilizable for small τ by any choice of C, P_n and Q-filter when the relative degree difference is greater than one.

Theorem 3: Under Assumption 1, suppose that $\text{r.deg}(P) \geq \text{r.deg}(P_n) + 2$. Then, for given P, there is no P_n, C, and Q such that, for sufficiently small $\tau > 0$, the closed-loop system is internally stable.

Proof: The proof is quite similar to that of Theorem 2. The root s^* converges to asymptotes at angles ϕ_i radiating out from the point $s = \sigma$ where

$$\phi_i = \frac{180^\circ + 360^\circ(i-1)}{m + 1 - m_s}, \quad i = 1, 2, \cdots, m + 1 - m_s$$

[3] Since the assumptions (i) and (ii) are equivalent to that $p_0(s)$ is Hurwitz, only the stable pole/zero cancellation between $p_0(s)$ and $sp_1(s)$ may happen.

[4] If $\alpha_{m_s}\beta_m < 0$, then we need to plot the negative root locus.

$$\sigma = \frac{\sum_{i=1}^{m} s_{1,i} - \sum_{i=1}^{m_s} s_{0,i}}{m+1-m_s}.$$

Note that $m+1-m_s = \text{r.deg}(P) - \text{r.deg}(P_n) + 1 \geq 3$, which implies that at least one of asymptotes has the angle ϕ_i such that $-90° < \phi_i < 90°$. Therefore, no matter where the point $s = \sigma$ is located, at least one s^* must enter the RHP for sufficiently small $\tau > 0$. ■

Now, we interpret the condition (5) in terms of P_n, P, and C. To this end, we denote the poles and the zeros of $P(s)$, and those of $P_n(s)$ by p_i, z_i, and p_i^n, z_i^n, respectively. For some nonnegative integers $l_p, k_p, l_n, k_n, l_c, k_c$, let

$$\begin{aligned}
D(s) &= (s^{l_p} + a_{p,1}s^{l_p-1} + \cdots + a_{p,l_p}),\\
N(s) &= K_p(s^{k_p} + b_{p,1}s^{k_p-1} + \cdots + b_{p,k_p}),\\
D_n(s) &= (s^{l_n} + a_{n,1}s^{l_n-1} + \cdots + a_{n,l_n}),\\
N_n(s) &= K_n(s^{k_n} + b_{n,1}s^{k_n-1} + \cdots + b_{n,k_n}),\\
D_c(s) &= (s^{l_c} + a_{c,1}s^{l_c-1} + \cdots + a_{c,l_c}),\\
N_c(s) &= K_c(s^{k_c} + b_{c,1}s^{k_c-1} + \cdots + b_{c,k_c}),
\end{aligned}$$

where K_p, K_n, and K_c denotes the high frequency gains of P, P_n, and C, respectively.

Theorem 4: Under Assumption 1, suppose that $\text{r.deg}(P_n) = \text{r.deg}(P) - 1$ and $\alpha_{m_s}\beta_m > 0$. Then, we have the followings: (i) If $\text{r.deg}(P_nC) \geq 2$, then (5) is equivalent to

$$\sum_{i=1}^{k_n} z_i^n - \sum_{i=1}^{k_p} z_i - \sum_{i=1}^{l_n} p_i^n + \sum_{i=1}^{l_p} p_i < 0. \tag{6}$$

(ii) If $\text{r.deg}(P_nC) = 1$, then (5) is equivalent to

$$\sum_{i=1}^{k_n} z_i^n - \sum_{i=1}^{k_p} z_i - \sum_{i=1}^{l_n} p_i^n + \sum_{i=1}^{l_p} p_i + K_nK_c < 0. \tag{7}$$

Proof: **(i)** It follows from $\text{r.deg}(P_nC) \geq 2$ that $\deg(ND_cD_n) \geq \deg(NN_cN_n) + 2$. In addition, since $\text{r.deg}(PC) = \text{r.deg}(P_nC) + 1 \geq 3$, we have $\deg(N_nD_cD) \geq \deg(N_nN_cN) + 3$. Thus, since $m_s = k_p + l_n + l_c$ and $m = k_n + l_p + l_c$, it follows that

$$\begin{aligned}
p_0(s) &= K_p[s^{m_s} + (a_{n,1} + a_{c,1} + b_{p,1})s^{m_s-1} + \cdots],\\
p_1(s) &= K_n[s^m + (a_{p,1} + a_{c,1} + b_{n,1})s^{m-1} + \cdots],
\end{aligned}$$

which implies

$$\begin{aligned}
\alpha_{m_s} &= K_p,\ \alpha_{m_s-1} = K_p(a_{n,1} + a_{c,1} + b_{p,1}),\\
\beta_m &= K_n,\ \beta_{m-1} = K_n(a_{p,1} + a_{c,1} + b_{n,1}).
\end{aligned}$$

Thus, (5) becomes

$$a_{n,1} + b_{p,1} - a_{p,1} - b_{n,1} < 0,$$

which leads to

$$\sum_{i=1}^{k_n} z_i^n - \sum_{i=1}^{k_p} z_i - \sum_{i=1}^{l_n} p_i^n + \sum_{i=1}^{l_p} p_i < 0.$$

(ii) Note that $\deg(ND_cD_n) = \deg(NN_cN_n) + 1$ and $\deg(N_nD_cD) = \deg(N_nN_cN) + 2$. Thus, it can be shown that

$$\begin{aligned}
p_0(s) &= K_p[s^{m_s} + (a_{n,1} + a_{c,1} + b_{p,1}\\
&\qquad + K_nK_c)s^{m_s-1} + \cdots],\\
p_1(s) &= K_n[s^m + (a_{p,1} + a_{c,1} + b_{n,1})s^{m-1} + \cdots],
\end{aligned}$$

which implies

$$\begin{aligned}
\alpha_{m_s} &= K_p,\ \alpha_{m_s-1} = K_p(a_{n,1} + a_{c,1} + b_{p,1} + K_nK_c),\\
\beta_m &= K_n,\ \beta_{m-1} = K_n(a_{p,1} + a_{c,1} + b_{n,1}).
\end{aligned}$$

Thus, (5) becomes

$$a_{n,1} + b_{p,1} + K_nK_c - a_{p,1} - b_{n,1} < 0,$$

which is equivalent to

$$\sum_{i=1}^{k_n} z_i^n - \sum_{i=1}^{k_p} z_i - \sum_{i=1}^{l_n} p_i^n + \sum_{i=1}^{l_p} p_i + K_nK_c < 0.$$

■

According to Theorem 4, as poles (zeros, respectively) of P_n are placed further right (left, respectively), it becomes more beneficial for the robust stabilization. However, this may impose quite a burden to the design of $C(s)$, since the control of unstable plant is usually more difficult than that of stable plant. In the meanwhile, there exists a difference between the case of $\text{r.deg}(P_nC) = 1$ and of $\text{r.deg}(P_nC) \geq 2$. Robust stabilization for the case where $\text{r.deg}(P_nC) = 1$ becomes more difficult than the case where $\text{r.deg}(P_nC) \geq 2$ since K_nK_c is usually positive. In other words, the design of C needs to take into consideration not only the stabilization of $\frac{P_nC}{1+P_nC}$ but also (7). As usual, the larger K_n is necessary for faster transient response and the smaller steady state error, which implies that (7) may impose a limitation on the performance of the closed-loop system.

4. AN ILLUSTRATIVE EXAMPLE

Consider the following system

$$P(s) = \frac{1}{(s+2)(\epsilon s+1)},$$

where ϵ is a nonnegative constant. We first consider the case where the relative degrees of P_n is the same as that of P. To this end, we assume $\epsilon = 0$. The DOB controller is designed using

$$P_n(s) = \frac{1}{(s+2)},\ C(s) = \frac{5}{s},\ Q(s) = \frac{1}{\tau s+1},$$

where $\tau = 0.01$. Fig. 2 shows the step responses of three cases: Nominal closed-loop system in the absence of disturbance ('Nominal response'), nominal closed-loop system in the presence of disturbance $d(t) = \sin(2\pi t)$ ('DOB OFF'), and the DOB controller turned on at $t =$

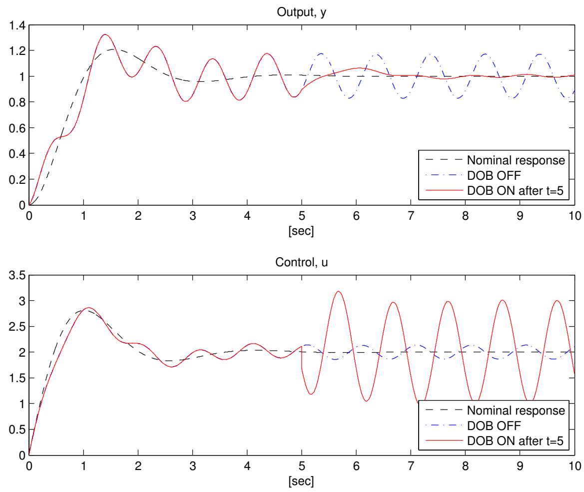

Fig. 2 Simulation results of $P(s) = \frac{1}{s+2}$. Nominal system in the absence of disturbance ('Nominal response'), nominal system in the presence of disturbance $d(t) = \sin(2\pi t)$ ('DOB OFF'), and DOB controller turned on at $t = 5$ in the presence of disturbance $d(t)$ ('DOB ON').

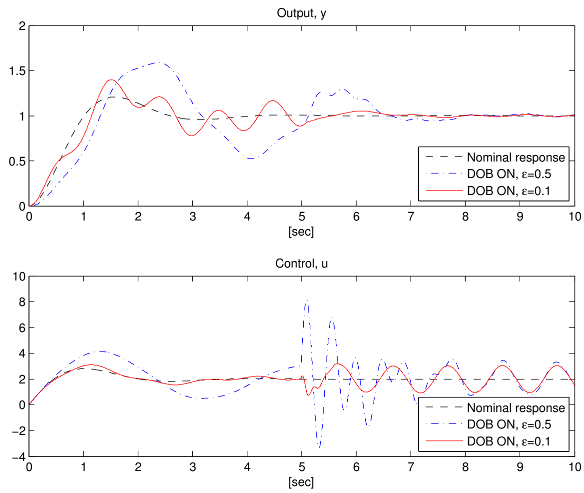

Fig. 3 Simulation results of $P(s) = \frac{1}{(s+2)(\epsilon s+1)}$. Nominal system (i.e., $\epsilon = 0$) in the absence of disturbance ('Nominal response'), DOB controller for $\epsilon = 0.5$ and $d(t) = \sin(2\pi t)$, and DOB controller for $\epsilon = 0.1$ and $d(t) = \sin(2\pi t)$. In both cases, DOB is switched on at $t = 5$.

5 in the presence of disturbance $d(t)$ ('DOB ON after $t = 5$'). From the figure, it is seen that the plant output of 'DOB ON' approaches that of 'Nominal response' as time goes on although there exists an input disturbance $d(t)$. This confirms the performance recovery equation (3).

Now, we study the case where $\text{r.deg}(P_n) = \text{r.deg}(P) - 1$. To this end, we consider two cases that $\epsilon = 0.5$ and $\epsilon = 0.1$, i.e., $P(s) = \frac{1}{(s+2)(0.5s+1)}$ and $P(s) = \frac{1}{(s+2)(0.1s+1)}$. We assume that both plants are approximated to $\frac{1}{s+2}$, from which the nominal plant is chosen as $P_n(s) = \frac{1}{s+2}$. The DOB controller is chosen as the same as in the above, which shows that the con-

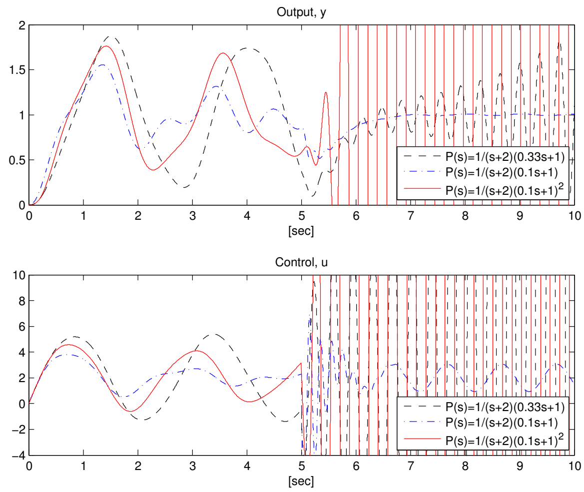

Fig. 4 Simulation results of plants that violate conditions of Theorems 2 and 4.

dition (6) is satisfied. Simulation results are shown in Fig. 3, where the dashed line represents a nominal system in the absence of disturbance. On the other hand, the dash-dotted line stands for a case where $\epsilon = 0.5$ and $d(t) = \sin(2\pi t)$ while the solid line for $\epsilon = 0.1$ and $d(t) = \sin(2\pi t)$. In both cases, DOB is switched on at $t = 5$. After the DOB is switched on, both plant outputs converge to that of 'Nominal response' as time goes by although there exists an input disturbance $d(t)$.

Finally, in order to appreciate conditions of Theorems 2 and 3, we consider the following three plants: $P_1(s) = \frac{1}{(s+2)(0.33s+1)}$, $P_2(s) = \frac{1}{(s+2)(0.1s+1)}$, and $P_3(s) = \frac{1}{(s+2)(0.1s+1)^2}$. We assume that the nominal plant is chosen as $P_n(s) = \frac{1}{s+6}$ rather than $\frac{1}{s+2}$ and $C(s)$ is chosen as $C(s) = \frac{10}{s}$. Then, according to Theorems 2 and 3, $P_1(s)$ and $P_3(s)$ are not stabilizable no matter how $C(s)$ and $Q(s)$ are chosen to design the DOB controller. Fig. 4 shows the simulation results of the DOB controller with $Q(s) = \frac{1}{\tau s+1}$. It is seen that the plant output of $P_2(s)$ is recovered to nominal one while those of $P_1(s)$ and $P_3(s)$ show very large oscillations and results in instability. Here, the instability of $P_3(s)$ is somewhat surprising because a well-known idea of designing controller is to approximate given plant to a simpler one by neglecting the fast dynamics, if any. However, it is seen from the figure that, as far as the DOB controller is concerned, this approximation should be made very carefully so that the difference between the relative degree of real plant and that of the approximated one must be less than two.

5. CONCLUSION

In this paper, we study the stability of the DOB control system when the relative degree of the plant is greater than that of the nominal model. Our research results show the following new results: 1) When the relative degree difference is one, robust stabilization can be achieved against arbitrarily large uncertainties. However, on the contrary to the same relative degree case, not only Q-filter but also P_n should be carefully chosen to achieve the goal. 2) When the relative degree difference is greater than one, robust stabilization is not possible no matter how Q-filter and P_n are chosen. Therefore, in order to achieve robust stabilization using DOB, P_n should be carefully selected such that the relative degree difference is not greater than one. The future research direction will be the stability analysis for the case where the Q-filter has a general form (2) rather than the simple form in Assumption 1.

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REFERENCES

[1] K. Ohishi, K. Ohnishi, and K. Miyachi, "Torque-speed regulation of dc motor based on load estimation method," In *Proc. of the International Power Electronics Conference*, 1983.

[2] H.S. Lee and M. Tomizuka, "Robust motion controller design for high-accuracy positioning systems," *IEEE Trans. Ind. Electron.*, vol. 43, pp. 48–55, 1996.

[3] C.J. Kempf and S. Kobayashi, "Disturbance observer and feedforward design for a high-speed direct-drive positioning table," *IEEE Trans. on Contr. Sys. Tech.*, vol. 7, no. 5, pp. 513–526, 1999.

[4] E. Schrijver and J. van Dijk, "Disturbance observers for rigid mechanical systems: equivalence, stability, and design," *J. of Dynamic Systems, Measurement, and Control*, vol. 124, pp. 539–548, 2002.

[5] J. Back and H. Shim, "Adding robustness to nominal output feedback controllers for uncertain nonlinear systems: A nonlinear version of disturbance observer," *Automatica*, vol. 44, pp. 2528–2537, 2008.

[6] J. Yi, S. Chang and Y. Shen, "Disturbance-Observer-Based Hysteresis Compensation for Piezoelectric Actuators," *IEEE/ASME Trans. Mechatronics.*, vol. 14, pp. 456–464, 2009.

[7] J. Bang, H. Shim and S.K. Park, "Robust Tracking and Vibration Suppression for a Two-Inertia System by Combining Backstepping Approach With Disturbance Observer," *IEEE Trans. Industrical Electronics.*, vol. 57, pp. 3197–3206, 2010.

[8] B. Yao, M. Al-Majed and M. Tomizuka, "High-performance robust motion control of machine tools: an adaptive robust control approach and comparative experiments," *IEEE/ASME Trans. Mechatronics*, vol. 2, pp. 63–76, 1997.

[9] Y. Choi, K. Yang, W.K. Chung, H.R. Kim, and I.H. Suh, "On the robustness and performance of disturbance observers for second-order systems," *IEEE Trans. Automat. Contr.*, vol. 48, pp. 315–320, 2003.

[10] B.K. Kim, W.K. Chung, "Advanced disturbance observer design for mechanical positioning systems," *IEEE Trans. Industrical Electronics.*, vol. 50, pp. 1207–1216, 2003.

[11] H. Shim and N.H. Jo, "An almost necessary and sufficient condition for robust stability of closed-loop systems with disturbance observer," *Automatica*, vol. 45, pp. 296–299, 2009.

[12] N.H. Jo, H. Shim, and Y.I. Son, "Disturbance Observer for Non-minimum Phase Linear Systems and Its Application to Autopilot Design," *International Journal of Control, Automation, and Systems*, vol. 8, pp. 994–1002, 2010.

[13] G.F. Franklin, J.D. Powell and A. Emami-Naeini, *Feedback Control of Dynamic Systems. 4th ed.*, Prentice Hall, 2002.